**Urban Prairie Waldorf School**
**Milestones for Mathematics**

### Aims for UPWS Mathematics Curriculum

- To sense, imagine and value the beauty of mathematical form within the natural, artistic and built environment and experience the joy of mathematical discoveries, processes and thinking.
- To develop mathematical capacities and skills for daily life in family and community.
- To prepare students for the vocational landscape with its increasingly technical and scientific approach through developing mathematical capacities skills that can be applied to any area of furtherstudy.
- To enhance the capacity of mathematical and creative thinking and develop confidence in the reliability and accuracy of the students' own thought processes and problem solving abilities.
- To develop the students' skills of discernment and so prepare them to make informed and ethical decisions about issues that involve mathematical knowledge in finance and business.
- To connect the students to the historical stream of human endeavor in mathematics and the process of growth in culture.
- To develop a high level of fluency in computational skills before any extension into thetechnological realm.
- To use insight in mathematical thought to examine and reflect on human social, scientific, philosophical or ethical dilemmas.

### UPWS Standards for Mathematical Instruction

These milestones embody Urban Prairie's research into mathematics as it lives and thrives at our school. As with any living entity, this document will transform and adapt to best meet its purpose: To serve as a tool for the faculty to call forth a comprehensive, profound, living, and elegant relationship to mathematics in our students. Secondarily, we aim for these milestones to serve the greater educational community (both Waldorf and beyond) as a healthy, conscious, functional, and realizable model for the mathematics curriculum in the early and middle grades.

This document sets forth various milestones for the children in each grade. While it is necessary for the content descriptions to be covered, teachers may use their professional judgment concerning the needs of their grade and the children in front of them. Content may be recombined or reallocated into different lessons, contexts, or time frames. In this way we ensure the freedom of the teachers and enable them to work with their children in the most efficient and appropriate way.

Note that Urban Prairie milestones are cross-referenced to corresponding standards in the Common Core State Standards Initiative. The grade level is indicated after the word 'content'. For example, CCSS.Math.Content.K.OA.A.1 would be Kindergarten and CCSS.Math.Content.3.MD.A.1 would be a grade 3 standard.

The following sections provide the context within which these milestones are brought, including UPWS's perspectives on child studies, the inner development of the teacher, and expectations of our teachers within the mathematics curriculum. Together, these perspectives are the foundations upon which we strive to expand the term "academics" to meet an evolving time.

**The child as primary text**

In terms of the understanding of child development, UPWS looks towards its faculty and children as the primary source. One of the strongest hallmarks of Waldorf education is the close and careful observation of the children placed in our care and trust. Through this observation, we are able to come to as complete as possible a picture of the internal and external development of a person throughout childhood. Each age and grade has its defining characteristics, and by working in harmony with these forces of transformation and the enigmas they present, we are able to chart a course that is, at its core, in harmony with the children themselves. This is no easy task, and it demands a great deal of the teacher and the supporting faculty. Our ongoing task is to form an appropriate response to the riddles and questions each child places in front of us. Many aspects of the child, whether of a constitutional or more subtle, "soul," nature, constantly point us in the right direction. But as any teacher or parent knows, these clues are not always easily identified.

**Supported inner development of teachers**

The work of the faculty, through child studies, personal inner development, and the consistent review of our curriculum and methodologies, ensures that we are being as clear and concise as possible in defining the questions underlying how the children respond to the curriculum as it's brought to them and the subsequent answers to these questions. For this reason alone, we realize the utmost importance placed on the inner development of the teacher. At UPWS, each teacher undertakes a meditative practice of their choosing, allowing us to come together, regardless of tradition or culture, to work as a supportive whole in the development of our students. A proper meditative life allows for enhanced clarity, harmonious communication, and a space to open up from which we are able to work. Thankfully, another hallmark of Waldorf education is the close collaboration between schools, on a national and international level. UPWS is able to tap into a wellspring of knowledge and experience that extends far beyond the walls of our school, and we are ever grateful for the aid and insights offered to us by experienced educators around the world. We also draw strongly from the indications of Rudolf Steiner as they apply to the development of the child. Steiner's observations inform what has become much of the foundation of modern Waldorf education. In working with them, deconstructing them, and suiting them to our own unique situation, we seek to provide the most healthy and thorough education as possible to our students.

**Expectations in teaching**

An education is only as good as the teachers who bring it. A robust and vigorous math curriculum calls for educators who are able to see math in the world around them and recognize the beauty and truth inherent therein. We strive to approach the subject with an aesthetic eye, instilling a sense of artistry and wonder in the children themselves. UPWS educators have a profound conceptual understanding of elementary mathematics and the ability to provide a foundation of that conceptual structure in our students. A profound understanding of mathematics has breadth, depth, and thoroughness. Breadth of understanding is the capacity to connect a topic with topics of similar or less conceptual power. Depth of understanding is the capacity to connect a topic with those of greater conceptual power. Thoroughness is the capacity to connect all topics. Without such an approach, the education is fragmented. Urban Prairie calls upon its educators to fully and deeply understand the mathematical principles and theory behind the curriculum. A deep penetration into the subject, far beyond what is brought to the children, allows one to understand how best to approach these milestones and bring them in a way which is meaningful, efficient, and effective. UPWS teachers are mathematically

confident and actively participate in current mathematics teaching reform, as research into current and emerging methods of math education is necessary for a comprehensive understanding of the subject.

Beginning in grade 1 a UPWS teacher connects the mathematical curriculum to the world surrounding us. She or he also reveals connections among mathematical concepts and processes to students. He or she appreciates different aspects of an idea and various approaches to a solution, as well as their advantages and disadvantages—and is able to provide coherent explanations of these various aspects and approaches. An effective teacher is aware of the simple but powerful basic ideas of mathematics and tends to revisit and reinforce them. He or she has a thorough understanding of the whole elementary mathematics curriculum and takes advantage of an opportunity to review concepts that students have previously studied or to lay the groundwork for a concept to be studied later. In this way, a teacher is able to instill a sense of confidence, ease, and appreciation for mathematics in the students.

At UPWS, new math concepts are brought through image and imaginative pictures. This is especially true for grades one through four. Only after working with a particular image and its attendant or inherent meanings and relationships, is a shift made to the conceptual and abstract. It is in this way that the child forms a living relationship to the content. It is a relationship that allows for growth from within. Applied with an understanding of a child's physiological, cognitive, and spiritual development, we are able to provide an education that is efficient, harmonious, and in accord with the children in front of us.

Efficient teaching of mathematics employs movement, rhythm, tempo, and joy. Music, movement and their related elements are deeply connected with mathematics, as they demonstrate aspects of spatial awareness, reasoning, sequencing, counting, patterning, and one-to-one correspondence. Using a rhythmic hand-clapping beat to learn times tables takes advantage of the fact that we typically process steady beat in the same area of the brain related to attention. Hand clapping and rhythmic movements also cross midlines, or our sagittal, coronal, and axial planes. Integration and effortless crossing of these bodily planes are paramount for ease of computation in such mathematical processes as long division, solving and checking equations, fractions, and many more. UPWS teachers strive to incorporate these ancillary elements of mathematics into daily practice in an effort to support the subject through its underlying components while working with the skills. In this way mathematics--especially in the younger grades--flows from imagination, rhythm, and movement to cognitive capacities.

In mathematics, it must be understood that we are working with truth: pure and elegant. Within the truths and in the relationships between the numbers we uncover archetypes. Archetypes are observable in the universe in myriad ways; they are the language of nature and natural order. In that sense, the imaginative pictures brought, no matter how inspired and creative, will somehow fall short of the sublimity they represent. The image, though useful, is only a tool to bring a class into concept in a healthy, living way. Once that relationship is established, the image is laid to rest, ready to be called upon if needed, but otherwise the children have moved on to pure concept and application. This is a subtle dance, wherein lies the artistic aspect of teaching. One freely determines where and when this takes place through careful observation of the children in an understanding of child development.

**Expanding the definition of "academic"**

UPWS offers a rich and deep academic experience, and in doing so we seek to develop the term "academic" and give it a modern perspective. Aware of its historical and cultural connotations, we aspire to bring it fully into the present times in a living and salient way. We see an academic education as one that involves the whole child and calls forth the best of their capabilities: intellectual,

physiological, social, and spiritual. Without each of these, the child is not whole, and in our efforts, we cannot discard one aspect without doing harm to or neglecting another. We see this as the new paradigm for modern education; UPWS strives to be at the forefront of this paradigm. This immense task requires the input and consideration of all stakeholders in our community in a courageous, open, fluid, and honest way. It is standard we strive to upkeep, as we see it as the cornerstone of our mission and purpose.

# Grade 1

*“Don't you know anything at all about numbers?"*
*"Well, I don't think they're very important," snapped Milo, too embarrassed to admit the truth.*
*"NOT IMPORTANT!" roared the Dodecahedron, turning red with fury. "Could you have tea for two without the two — or three blind mice without the three? Would there be four corners of the earth if there weren't a four? And how would you sail the seven seas without a seven?"*
*"All I meant was—" began Milo, but the Dodecahedron, overcome with emotion and shouting furiously, carried right on.*

*"If you had high hopes, how would you know how high they were? And did you know that narrow escapes come in all different widths? Would you travel the whole wide world without ever knowing how wide it was? And how could you do anything at long last," he concluded, waving his arms over his head, "without knowing how long the last was? Why, numbers are the most beautiful and valuable things in the world. Just follow me and I'll show you." He turned on his heel and stalked off into the cave.”*

— Norton Juster, *The Phantom Tollbooth*

The Grade 1 mathematics lesson calls upon the senses of movement, balance, and life. It awakens in the child the ability to experience forms and works on the bodily proprioceptive system and memory in a harmonizing, stimulating, and strengthening way. It refines the child’s fine motor skills and requires the child to visualize spatial relationships inwardly. Much practice is aimed at preventing children from becoming fixed in any extreme, instead enabling children to experience and move between polarities and to experience the central point of balance. Number sequences and patterns, times tables and number facts are all taught initially through coordinated speech, image and movement which imprint the learning more deeply. Each new skill or concept is brought through a story in which the actions and images clearly provide an enlivened experience of the process. This way the skills can be practiced with repetition that is new each day in its context and which builds layers of increased complexity naturally. The children will learn the qualitative aspects of the numbers from 1 to 12, be introduced to each of the four operations, and lay a solid foundation of number sense and arithmetical thinking.

**1) Operations and Algebraic Thinking**

a) Have a qualitative sense of the four processes: addition, subtraction, multiplication, division
   i) Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
      (1) Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (CCSS.Math.Content.K.OA.A.1)
      (2) Solve addition and subtraction word problems, and add and subtract within 24, e.g., by using objects or drawings to represent the problem. (CCSS.Math.Content.K.OA.A.2(Modified))
      (3) Decompose numbers less than or equal to 24 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). (CCSS.Math.Content.K.OA.A.3(Modified))
      (4) For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS.Math.Content.K.OA.A.4)
   ii) Understand and apply properties of operations and the relationship between addition and subtraction.
      (1) Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative*

*property of addition.)* (CCSS.Math.Content.1.OA.B.3)

iii) Add and subtract within 20
   (1) Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (CCSS.Math.Content.1.OA.C.5)
   (2) Add and subtract within 24, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (CCSS.Math.Content.1.OA.C.6(Modified))

iv) Represent and solve problems using addition and subtraction.
   (1) Use addition and subtraction within 24 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS.Math.Content.1.OA.A.1(Modified)
   (2) Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 24, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (CCSS.Math.Content.1.OA.A.2(Modified))
   (3) Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. (CCSS.Math.Content.1.NBT.C.5)

v) Work with addition and subtraction equations.
   (1) Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. (CCSS.Math.Content.1.OA.D.7)
   (2) Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ – 3, 6 + 6 = _*. (CCSS.Math.Content.1.OA.D.8)

vi) Work with equal groups of objects to gain foundations for multiplication.
   (1) Determine whether a group of objects has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (CCSS.Math.Content.2.OA.C.3)
   (2) Use addition to find the total number of objects arranged in groups or rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. (CCSS.Math.Content.2.OA.C.4)
   (3) Use multiplication of two one-digit numbers to solve word problems

vii) Represent and solve problems involving multiplication and division.
   (1) Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7*. (CCSS.Math.Content.3.OA.A.1)
   (2) Interpret whole-number quotients of whole numbers, including problems with remainders, e.g., interpret 24 ÷ 8 as the number of objects in each share when 24 objects are partitioned equally into 8 shares, or as a number of shares when 24 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 24 ÷ 8.* (CCSS.Math.Content.3.OA.A.2 )

viii) Understand properties of multiplication
   (1) Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.)* (CCSS.Math.Content.3.OA.B.5(Modified))

b) Continue and extend a pattern rhythmically, symbolically, numerically, alphabetically, in shape, or color.

**c)** Make use of rhymes and verses to remember algorithms

**2) Number Sense**

a) Experience a qualitative relationship with numbers 1--12.

b) See Gr1.3.a

c) See Gr1.3.b
d) See Gr1.3.c
e) See Gr1.3.d

**3) Counting and Cardinality**

a) Know number names and count the sequence.
   i) Count forward to 125 and backwards from 125 by ones and by tens. (CCSS.Math.Content.K.CC.A.1(Modified))
   ii) Count forward beginning from a given number within the known sequence (instead of having to begin at 1). (CCSS.Math.Content.K.CC.A.2)
   iii) Write numbers from 0 to 125. Represent a number of objects with a written numeral 0-125 (with 0 representing a count of no objects). (CCSS.Math.Content.K.CC.A.3(Modified))
b) Demonstrate knowledge of ordinal numbers.
c) Compare numbers
   i) Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.(CCSS.Math.Content.K.CC.C.6)
   ii) Compare two numbers between 1 and 125 presented as written numerals. (CCSS.Math.Content.K.CC.C.7(Modified))
d) Count to tell the number of objects. Understand the relationship between numbers and quantities; connect counting to cardinality. (CCSS.Math.Content.K.CC.B.4)
   i) When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (CCSS.Math.Content.K.CC.B.4a)
   ii) Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (CCSS.Math.Content.K.CC.B.4b)
   iii) Understand that each successive number name refers to a quantity that is one larger. (CCSS.Math.Content.K.CC.B.4c)
e) As a group, skip count by 2's, 3's, 4's, 5's, and 10's up to the $12^{th}$ multiple.

**4) Measurement and Data**

a) Estimate quantities and measurable attributes of objects
b) Describe and compare measureable attributes
   i) Describe measurable attributes of objects, such as number, length or weight with non standard units. Describe several measurable attributes of a single object. (CCSS.Math.Content.K.MD.A.1(Modified))
   ii) Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter*. (CCSS.Math.Content.K.MD.A.2)
c) Classify objects and count the number of objects in each category
   i) Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (CCSS.Math.Content.K.MD.B.3)
d) Measure lengths indirectly and by iterating length units.
   i) Order three objects by length; compare the lengths of two objects indirectly by using a third object. (CCSS.Math.Content.1.MD.A.1)
   ii) Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps*. (CCSS.Math.Content.1.MD.A.2)
e) Represent and interpret data
   i) Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (CCSS.Math.Content.1.MD.C.4)

5) **Number Operations in Base Ten**

**a)** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. (CCSS.Math.Content.1.NBT.A.1)

**6) Geometry**

a) Identify and describe basic geometric forms through movement, walking patterns, string formation, and drawing.
   i) Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. (CCSS.Math.Content.K.G.A.1)
   ii) Correctly name shapes regardless of their orientations or overall size. (CCSS.Math.Content.K.G.A.2)

b) Analyze, compare, create, and compose shapes.
   i) Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). (CCSS.Math.Content.K.G.B.4)
   ii) Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS.Math.Content.K.G.B.5)

c) Reason with shapes and their attributes
   i) Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. (CCSS.Math.Content.1.G.A.1)
   ii) Compose two-dimensional shapes (rectangles, squares, triangles, half-circles, and quarter-circles) to create a composite shape, and compose new shapes from the composite shape. (CCSS.Math.Content.1.G.A.2(Modified))

# Grade 2

“The thing I want you especially to understand is this feeling of divine revelation. I feel that this structure was "out there" all along. I just couldn't see it. And now I can! This is really what keeps me in the math game-- the chance that I might glimpse some kind of secret underlying truth, some sort of message from the gods.”

— Paul Lockhart, *A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*

The joy of learning continues to be built on a strong love of rhythm, recitation and movement as well as the colorful stories and beautiful geometric patterns. In the Second Grade the children are less dreamy. They can concentrate for longer periods. While still in transition developmentally from the preoperational stage, the beginnings of concrete operational thought are evident from age 7 onwards. This includes aspects of identity, reversibility and seriation (situating an object in a series). The children are able to identify, sequence and engage in the number processes. They see the relationship between the four operations and begin to look at practical strategies. They continue to deepen and broaden the foundation laid in Grade 1 and develop a greater understanding and fluency in working with numbers. The children will learn the concept of place value, develop an understanding of multi-digit addition and subtraction, and begin to work formally with the multiplication tables.

**1) Operations and Algebraic Thinking**

- **a)** Understand that addition and subtraction are opposite processes and use them to check work
- **b)** Represent and solve problems involving addition and subtraction
  - i) Use addition and subtraction within 1000 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS.Math.Content.2.OA.A.1(Modified)
- c) Add and subtract within 24
  - i) Fluently add and subtract within 48 using mental strategies. (CCSS.Math.Content.2.OA.B.2(Modified))
  - ii) Know from memory all sums of two numbers up to 24. (CCSS.Math.Content.2.OA.B.2(Modified))
- d) Represent and solve problems involving multiplication and division.
  - i) Use multiplication and division within 144 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS.Math.Content.3.OA.A.3)
  - **ii)** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?.* (CCSS.Math.Content.3.OA.A.4)
- **e)** Multiply and divide within 144.
  - **i)** Fluently multiply and divide within 144, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. (CCSS.Math.Content.3.OA.C.7(Modified))
  - **ii)** Know from memory all products with multiples of with 2, 3, 4, 5, and 10s and be familiar with products with multiples of 6, 7, 8, 9, 11, and 12.
- **f)** Solve Problems involving the four operations, and identify and explain patters in arithmetic.
  - **i)** Solve multi-step word problems and number journeys using the four operations, including problems with remainders. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS.Math.Content.3.OA.D.8 (Modified))
  - **ii)** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*. (CCSS.Math.Content.3.OA.D.9)

**g)** Make use of rhymes and verses to remember algorithms

**h)** Can continue and extend more complex patterns rhythmically, symbolically, numerically, alphabetically, in shape, or color.

**2) Number and Operations in Base Ten**

**a)** Understand Place Value

(1) Understand that the four digits of a four-digit number represent amounts of thousands, hundreds, tens, and ones; e.g., 2706 equals 2 thousands, 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: (CCSS.Math.Content.2.NBT.A.1(Modified)

(a) 10 can be thought of as a bundle of ten ones — called a "ten." (CCSS.Math.Content.1.NBT.B.2a)

(b) 100 can be thought of as a bundle of ten tens — called a "hundred." (CCSS.Math.Content.2.NBT.A.1a)

(c) 100 can be thought of as a bundle of ten hundreds – called a "thousand"

(d) The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS.Math.Content.1.NBT.B.2b)

**(e)** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). (CCSS.Math.Content.1.NBT.B.2c)

**(f)** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). (CCSS.Math.Content.2.NBT.A.1b)

**ii)** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. (CCSS.Math.Content.2.NBT.A.3)

**iii)** Count within 1000; skip-count by 5s, 10s, and 100s. (CCSS.Math.Content.2.NBT.A.2)

**iv)** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*. (CCSS.Math.Content.4.NBT.A.1)

**v)** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place.. (CCSS.Math.Content.4.NBT.A.2(Modified))

**b)** Use place value understanding and properties of operations to add and subtract

i) Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. (CCSS.Math.Content.1.NBT.C.4)

ii) Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS.Math.Content.1.NBT.C.6)

iii) Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS.Math.Content.2.NBT.B.5)

iv) Add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS.Math.Content.2.NBT.B.6)

v) Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS.Math.Content.2.NBT.B.7)

vi) Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. (CCSS.Math.Content.2.NBT.B.8)

vii) Explain why addition and subtraction strategies work, using place value and the properties of operations. (CCSS.Math.Content.2.NBT.B.9)

**c)** Use place value understanding and properties of operations to perform multi-digit arithmetic.
   i) Use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS.Math.Content.3.NBT.A.1)
   ii) Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (CCSS.Math.Content.3.NBT.A.2(Modified))
   iii) Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (CCSS.Math.Content.3.NBT.A.3)
   iv) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS.Math.Content.4.NBT.A.2)

**d)** Carrying and borrowing

**e)** Use the vertical format for all four processes

**f)** Use borrowing and carrying in simple problems

**g)** Can demonstrate using manipulative objects the concept of regrouping as used in carrying and borrowing.

**3) Counting and Cardinality**

**a)** Count forward and backward to 1,000

**b)** Read and write numbers to 10,000

**4) Geometry**

**a)** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (CCSS.Math.Content.2.G.A.1)

**b)** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS.Math.Content.2.G.A.2)

**c)** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. (CCSS.Math.Content.4.G.A.3)

# Grade 3

*"Were it not for number and its nature, nothing that exists would be clear to anybody either in itself or in its relation to other things...You can observe the power of number exercising itself ... in all acts and the thoughts of men, in all handicrafts and music."*

Pythagorean Philolaus (425 B.C.E.)

In Grade 3 knowledge, understanding, and skills are developed further by revising, enriching, and extending the strategies and computational work of the previous years. Simple division is consolidated through narrative and picture contexts as well as concrete work and written algorithms that bring the children closer to the mastery of the four operations. Fluency increases with continued practice. As the children explore possible solutions to story problems, particularly in the measurement and building topics, they deepen their number sense and mathematical thinking. The children thereby develop strong skills in using and adapting strategies and transferring understanding to new situations.This is a time when mathematics can be applied to the outside world that the child perceives objectively. Thus measurement of length, weight, capacity, time, and money provide a way of meeting the world and engaging in practical work such as building. The children also develop understanding and fluency with multi-digit multiplication and dividing to find quotients involving multi-digit dividends.

**1) Operations and Algebraic Thinking**

- **a)** Mentally solve two digit addition and subtraction problems.
- **b)** Use own words to rephrase and explain equations in complete sentences, as well as give examples from real life illustrating the equations.
- **c)** Use a variety of problem solving strategies; guess and check, solve a simpler problem, make a model or drawing, estimating and explain how they are used.
- **d)** Fluently use diverse vocabulary surrounding the four operations, e.g., quotient, sum, difference, product
- **e)** Use the four operations with whole numbers to solve problems.
  - **i)** Determine the unknown whole number in an equation relating three whole numbers up to 100. (CCSS.Math.Content.3.OA.A.4 (Modified))
  - **ii)** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS.Math.Content.4.OA.A.3)
- **f)** Long Division
- **g)** Generate and Analyze Patterns
  - **i)** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*. (CCSS.Math.Content.4.OA.C.5)
- **h)** Multiply and divide within 144.
  - **i)** By the end of Grade 3, know from memory all products of numbers 1 – 12.
- **i)** Can continue and extend more complex patterns rhythmically, symbolically, numerically, alphabetically, in shape, or color.

**2) Number and Operations in Base Ten**

- **a)** Generalize place value understanding for multi-digit whole numbers.
  - **i)** Use place value understanding to round multi-digit whole numbers to any place. (CCSS.Math.Content.4.NBT.A.3)

**b)** Use place value understanding and properties of operations to perform multi-digit arithmetic.
   i) Fluently add and subtract multi-digit whole numbers up to 10,000 using the standard algorithm with carrying and borrowing. (CCSS.Math.Content.4.NBT.B.4(Modified))
   **ii)** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS.Math.Content.4.NBT.B.5)
   **iii)** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS.Math.Content.4.NBT.B.6)

**3) Counting and Cardinality**
   **a)** Count forward and backward to 1,000
   **b)** Read and write numbers to 100,000,000,

**4) Measurement and Data**
   **a) Measure and estimate lengths in standard units.**
      **i)** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (CCSS.Math.Content.2.MD.A.1)
      **ii)** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. (CCSS.Math.Content.2.MD.A.2)
      **iii)** Estimate lengths using units of inches, feet, centimeters, and meters. (CCSS.Math.Content.2.MD.A.3)
      **iv)** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS.Math.Content.2.MD.A.4)
   **b) Solve problems involving measurement and estimation.**
      **i)** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS.Math.Content.3.MD.A.1)
      **ii)** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (CCSS.Math.Content.3.MD.A.2)
   **c) Relate addition and subtraction to length.**
      **i)** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS.Math.Content.2.MD.B.5)
      **ii)** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. (CCSS.Math.Content.2.MD.B.6)
   **d) Work with time and money.**
      **i)** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS.Math.Content.2.MD.C.7)
      **ii)** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS.Math.Content.2.MD.C.8)
   **e) Represent and interpret data.**
      **i)** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS.Math.Content.2.MD.D.9)
      **ii)** Draw a pictures and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (CCSS.Math.Content.2.MD.D.10)

**f) Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

**i)** Recognize area as an attribute of plane figures and understand concepts of area measurement. (CCSS.Math.Content.3.MD.C.5)

**(1)** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (CCSS.Math.Content.3.MD.C.5a)

(2) A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. (CCSS.Math.Content.3.MD.C.5b)

ii) Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). (CCSS.Math.Content.3.MD.C.6)

iii) Relate area to the operations of multiplication and addition. (CCSS.Math.Content.3.MD.C.7)

(1) Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. (CCSS.Math.Content.3.MD.C.7a)

(2) Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. (CCSS.Math.Content.3.MD.C.7b)

(3) Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. (CCSS.Math.Content.3.MD.C.7c)

(4) Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. (CCSS.Math.Content.3.MD.C.7d)

g) **Geometric measurement: recognize perimeter.**

i) Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS.Math.Content.3.MD.D.8)

h) Solve problems involving measurement and conversion of measurements.

i) Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* (CCSS.Math.Content.4.MD.A.1)

ii) Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS.Math.Content.4.MD.A.2)

iii) Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*. (CCSS.Math.Content.4.MD.A.3)

5) **Geometry**

a) Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (CCSS.Math.Content.1.G.A.2(Modified))

# Grade 4

*"All the secrets of nature are written down in that magnificent book which is continuously before our eyes. I refer to the universe. But we cannot understand it if we do not first learn the language and letters in which it is written. It is written in the language of mathematics and the letters are triangles, circles and other geometrical figures, without which we should find it impossible to understand a single word."*

*Galileo, 1546-1624*

A new confidence in their ability to meet the world characterizes this age, as children have boundless energy and an eagerness to look at the world and learn. Thus 4th Graders really begin to find their way into the world. They extend their practical skills with measurement and representation to mapping the local area. Their previous skills with measurement are now fully applied to a larger area. The ability to take a bird's eye view emerges, and they have a sense of scale, direction, and features by grid reference. The world of geometry opens further to freehand forms based on the circle and related to creative geometric patterning. They have the confidence to encompass the pictorial and concrete world of fractions; the whole number is broken apart just as they have lost the wholeness of earlier childhood. They can explore long multiplication and division as they stand on the boundary between the pictorial representation of previous years and the technical procedures of the later years. The fourth grader is able to solve a wide range of problems with the four operations, fractions, measurement, and mental arithmetic. Fourth graders draw freehand rosettes and forms arising from circle divisions and identify angles. They gather information and draw pie charts and picture graphs.

**1) Mathematics**

**a) Operations and Algebraic Thinking**

**i) Gain familiarity with factors and multiples**

**(1)** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (CCSS.Math.Content.4.OA.B.4(Modified))

**b) Number and Operations in Base Ten**

**i) Perform operations with multi-digit whole numbers**

(1) Fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS.Math.Content.5.NBT.B.5)

**(2)** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS.Math.Content.5.NBT.B.6)

**c) Number and Operations – Fractions**

i) Develop an understanding of fractions as numbers

(1) Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*. (CCSS.Math.Content.3.NF.A.1)

(2) Understand a fraction as a number on the number line; represent fractions on a number line diagram. (CCSS.Math.Content.3.NF.A.2 )

(a) Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (CCSS.Math.Content.3.NF.A.2a)

(b) Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b*

on the number line. (CCSS.Math.Content.3.NF.A.2b)

(3) Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (CCSS.Math.Content.3.NF.A.3)

(a) Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (CCSS.Math.Content.3.NF.A.3a)

(b) Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (CCSS.Math.Content.3.NF.A.3b)

(c) Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*. (CCSS.Math.Content.3.NF.A.3c)

**(d)** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (CCSS.Math.Content.3.NF.A.3d)

**ii)** Extend understanding of fraction equivalence and ordering

**(1)** Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (CCSS.Math.Content.4.NF.A.1)

**(2)** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (CCSS.Math.Content.4.NF.A.2)

**iii)** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

(1) Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. (CCSS.Math.Content.4.NF.B.3)

(a) Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (CCSS.Math.Content.4.NF.B.3a)

(b) Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. (CCSS.Math.Content.4.NF.B.3b)

(c) Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (CCSS.Math.Content.4.NF.B.3c)

(d) Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. (CCSS.Math.Content.4.NF.B.3d)

(2) Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (CCSS.Math.Content.4.NF.B.4)

(a) Understand a fraction $a/b$ as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*. (CCSS.Math.Content.4.NF.B.4a)

(b) Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* (CCSS.Math.Content.4.NF.B.4b)

**(c)** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many*

*pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* (CCSS.Math.Content.4.NF.B.4c)

**iv) Use equivalent fractions as a strategy to add and subtract fractions**

(1) Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* (CCSS.Math.Content.5.NF.A.1)

**(2)** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*. (CCSS.Math.Content.5.NF.A.2)

**v) Apply and extend understandings of multiplication and division to multiply and divide fractions**

(1) Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (CCSS.Math.Content.5.NF.B.3)

(2) Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (CCSS.Math.Content.5.NF.B.4)

(a) Interpret the product ($a/b$) × $q$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* (CCSS.Math.Content.5.NF.B.4a)

(b) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (CCSS.Math.Content.5.NF.B.4b)

(3) Interpret multiplication as scaling (resizing), by: (CCSS.Math.Content.5.NF.B.5)

(a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (CCSS.Math.Content.5.NF.B.5a)

(b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying $a/b$ by 1. (CCSS.Math.Content.5.NF.B.5b)

(4) Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS.Math.Content.5.NF.B.6)

(5) Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (CCSS.Math.Content.5.NF.B.7)

(a) Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3*. (CCSS.Math.Content.5.NF.B.7a)

(b) Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4*. (CCSS.Math.Content.5.NF.B.7b)

**(c)** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* (CCSS.Math.Content.5.NF.B.7c)

**d) Measurement and Data**

**i)** Represent and Interpret Data

(1) Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets*. (CCSS.Math.Content.3.MD.B.3)

**(2)** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. (CCSS.Math.Content.3.MD.B.4)

**(3)** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection*. (CCSS.Math.Content.4.MD.B.4)

**(4)** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical containers, find the amount of liquid each container would contain if the total amount in all the containers were redistributed equally*. (CCSS.Math.Content.5.MD.B.2)

**e) Geometry**

**i)** Reason with shapes and their attributes

**(1)** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS.Math.Content.1.G.A.3)

**(2)** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape*. (CCSS.Math.Content.3.G.A.2)

# Grade 5

*"The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated ... The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of antiquity, Archimedes and Apollonius."*
— Pierre Simon De Laplace

In grade 5, the students have a desire to be challenged and to improve their skills. They consolidate earlier learning and are conscious of wanting to reach a level of proficiency. The students become increasingly competent in mathematical skills and independent of pictorial representation. They solve problems, chose strategies and work with decimals. In geometry they are able to complete complex constructions and begin to work with instruments. The beauty of form in geometry speaks to the children. The more accurate their constructions, the greater their aesthetic quality. They are interested in discovering the properties of numbers that can still fill them with wonder. In links to other learning areas, study of ancient cultures affords an opportunity to integrate mathematical learning through the experience of number, geometry and measurement in Ancient India, Persia, Babylon, Egypt and Greece.

**1) Operations and Algebraic Thinking**
- **a)** Practice mental arithmetic in relation to decimals.
- **b)** Extend understanding of place value to billions.
- **c)** Fluently divide and multiply fractions.
- **d)** Use different number systems.

**2) Number and Operations in Base Ten**
- a) Understand the place value system
  - i) Read, write, and compare decimals to thousandths. (CCSS.Math.Content.5.NBT.A.3)
    - (1) Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). (CCSS.Math.Content.5.NBT.A.3a)
    - (2) Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (CCSS.Math.Content.5.NBT.A.3b)
  - ii) Use place value understanding to round decimals to any place. (CCSS.Math.Content.5.NBT.A.4)
- b) Perform operations with multi-digit whole numbers and with decimals to thousandths.
  - i) Add, subtract, multiply, and divide decimals to thousandths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (CCSS.Math.Content.5.NBT.B.7(Modified))

3) Number and Operations – Fractions
- a) Understand decimal notation for fractions, and compare decimal fractions.
  - i) Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. (CCSS.Math.Content.4.NF.C.6)
  - ii) Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (CCSS.Math.Content.4.NF.C.7)

4) Measurement and Data
- a) Convert like measurement units within a given measurement system

i) Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. (CCSS.Math.Content.5.MD.A.1)

b) Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition
   i) Recognize volume as an attribute of solid figures and understand concepts of volume measurement. (CCSS.Math.Content.5.MD.C.3)
      (1) A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. (CCSS.Math.Content.5.MD.C.3a)
      (2) solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. (CCSS.Math.Content.5.MD.C.3b) A
   ii) Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. (CCSS.Math.Content.5.MD.C.4)
   iii) Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. (CCSS.Math.Content.5.MD.C.5)
      (1) Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (CCSS.Math.Content.5.MD.C.5a)
      (2) Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. (CCSS.Math.Content.5.MD.C.5b)
      (3) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. (CCSS.Math.Content.5.MD.C.5c)

**5) Geometry**

a) Reason with shapes and their attributes.
   **i)** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (CCSS.Math.Content.3.G.A.1)

**b)** Draw and identify lines and angles, and classify shapes by properties of their lines and angles
   i) Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (CCSS.Math.Content.4.G.A.1)
   **ii)** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. (CCSS.Math.Content.4.G.A.2)

c) Measure with a protractor.

d) Make simple constructions with compass, protractor and ruler and highlight the forms with color.

e) Visualize and solve problems requiring geometric constructions.